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Short Writeup P08

For N1 and N2, we used e = 0.1.

For N3, we used e = 0.01.

Normals N1 and N2 are equal to each other, while N3 is not necessarily equal to either of the other estimates. It is possible for N3 to be equal to the other normal estimates, as in the instance of a perfectly flat Coon's Patch, but it is not always the case. The reason for this is that it uses different points for the tangents than the other two estimates. The relation between the corresponding points and the corresponding tangents is based on the Coon's Patch rather than the normal estimations, so we actually cannot derive any information about N3 from N1 or N2 and vice versa. Of course we can assume that they should be very similar.

We can prove that N1 = N2 using algebra. We can declare points A, B, C, D, and V such that:

A = (Ax, Ay, Az)

B = (Bx, By, Bz)

C = (Cx, Cy, Cz)

D = (Dx, Dy, Dz)

V = (Vx, Vy, Vz)

Consequently,

AC = (Cx - Ax, Cy - Ay, Cz - Az)

BD = (Dx - Bx, Dy - By, Dz - Bz)

VA = (Ax - Vx, Ay - Vy, Az - Vz)

VB = (Bx - Vx, By - Vy, Bz - Vz)

VC = (Cx - Vx, Cy - Vy, Cz - Vz)

VD = (Dx - Vx, Dy - Vy, Dz - Vz)

If we compute N1 = VAxVB + VBxVC + VCxVD + VDxVA and simplify that gives us

N1 = <AyBz - AzBy + ByCz - BzCy + CyDz - CzDy + DyAz - DzAy,

AzBx - AxBz + BzCx - BxCz + CzDx - CxDz + DzAx - DxAz,

AxBy - AyBx + BxCy - ByCx + CxDy - CyDx + DxAy - DyAx>

This is equivalent to ACxBD, so N1 and N2 are always equal.

Because N3 uses a much smaller e than N1 and N2, it will in most cases produce an estimation of the normal that is closer to the actual normal than N1 or N2 are. The tangent lines VA, VB, VC, and VD will most likely become closer to the actual tangents at V in the corresponding directions as e approaches 0. For this reason N3 is in most cases the best estimate of the actual normal at V.